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The irreducible string and an infinity of additional constants of motion in a deposition–evaporation model on a line

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Abstract. We study a model of stochastic deposition–evaporation with recombination, of three species of dimers on a line. This model is a generalization of the model recently introduced by Barma *et al* to $q \geqslant 3$ states per site. It has an infinite number of constants of motion, in addition to the infinity of conservation laws of the original model which are encoded as the conservation of the irreducible string. We determine the number of dynamically disconnected sectors and their sizes in this model exactly. Using the additional symmetry we construct a class of exact eigenvectors of the stochastic matrix. The autocorrelation function decays with different powers of t in different sectors. We find that the spatial correlation function has an algebraic decay with exponent 3/2, in the sector corresponding to the initial state in which all sites are in the same state. The dynamical exponent is non-trivial in this sector, and we estimate it numerically by exact diagonalization of the stochastic matrix for small sizes. We find that in this case $z = 2.39 \pm 0.05$.

1. Introduction

Recently a very interesting stochastic model with deposition and evaporation processes has been introduced by Barma *et al* [1, 2]. In this model one deposits atoms onto k adjacent vacant sites of a d-dimensional lattice and evaporates atoms from any k adjacent occupied sites, with specific rates for deposition and evaporation. The cases $k = 1$ and $k = 2$ are exactly solvable, the former being trivial, and the latter being equivalent to the ferromagnetic Heisenberg spin-$\frac{1}{2}$ chain. For $k \geqslant 3$, on a linear chain of length L, the phase space of this k-mer model consisting of the 2^L possible configurations is found to break up into an exponentially large number of dynamically disconnected sectors [1, 2]. This may be understood as being due to the existence of an infinite number of independent conserved quantities in this model [3]. These conservation laws also give rise to a wide range of relaxation behaviour [4]. For example the density–density autocorrelation function shows different power law decays in different sectors ($t^{-1/4}, t^{-1/2}, t^{-0.59}$) and even a stretched exponential decay in some sectors.

For $k \geqslant 3$, though there exist an infinite number of constants of motion, these still have not enabled us to get a full solution of the model so far. In this case the number of sectors and their sizes in the steady state has been calculated exactly, but the dynamics are understood only qualitatively. The quantum Hamiltonian corresponding to the stochastic

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matrix has k-body interaction terms which makes the problem difficult to tackle analytically. Thus it is worthwhile studying models which show the same qualitative features and are at the same time more tractable analytically.

With this motivation we define a variant of the k-mer deposition–evaporation model with q states per site. This model for $q = 2$ corresponds to the dimer deposition evaporation model studied by Barma *et al.* For $q \geqslant 3$ this model shares many qualitative features with the trimer model, like the existence of an infinity of conservation laws encoded as the conservation law of the 'irreducible string' [3], and different power law decay of the autocorrelation function in different sectors. However, our model has the two advantages: it has only two-body interaction and it has an additional discrete symmetry group of very large order. These properties make this model more tractable analytically than the k-mer model. In fact, one can get many exact eigenvectors of the stochastic matrix in our model very easily. However, we have not succeeded so far in solving this model completely. For simplicity we study only $q = 3$ case in this paper, for higher q results will be qualitatively the same.

This paper is organized as follows. In section 2 we define our model and write the stochastic evolution operator in the form of the Hamiltonian of a quantum many-body system of three species of particles. In section 3 we show that the vector space consisting of 3^L configurations breaks up into an exponentially large number of dynamically disconnected sectors, and also calculate their sizes. In section 4 we describe the additional symmetries of this model and study the resulting further decomposition of the phase space formed by all the configurations belonging to a sector into subspaces. We construct a class of exact eigenvectors in the next section. In section 6 we study the decay of autocorrelation function in various sectors. As in the case of trimer model [4], we find sector-dependent decay. The spatial correlation function in the steady state for the sector corresponding to an initial condition of all sites in the same state is calculated in section 7. It has power law decay with exponent 3/2. A numerical diagonalization study of the stochastic matrix to find the dynamical exponent in this sector is presented in section 8.

2. Definition of the model

Our model is defined as follows. At each site of a one-dimensional lattice, there is a spin variable q_i which can be in any of the three states a, b or c. These spin variables undergo a stochastic time evolution given by the following rule: any pair of adjacent spins, which are in the same state, can flip together to any of the other two states with some specified rates for the transitions. For example an aa pair can flip to become either a bb pair or a cc pair. This process can be thought of as the evaporation of an aa dimer and immediate deposition of a bb or cc dimer at that place. We shall call this model the dimer deposition–evaporation model (DDE model). This is a special case of the general reaction–diffusion process ($a + b \rightleftharpoons c + d$) recently studied by Dahmen [5]. We consider only the case of equal rates for all the transitions in this paper.

Any configuration C on the lattice can be represented by the basis vector $|q_1, q_2, \ldots, q_L\rangle$, in a vector space of dimension 3^L. If $P(C, t)$ denotes the probability of the configuration C at time t, then the master equation describing the evolution of these probabilities can be written as

$$\frac{\partial}{\partial t}|P(t)) = \hat{W}|P(t)) \tag{1}$$

where $|P(t)) = \sum_C P(C, t)|C)$ and $\hat{W}$ is the stochastic matrix.

We can write $\hat{W}$ in the form of a quantum Hamiltonian as follows. Consider a quantum mechanical system of three species of particles on a one-dimensional lattice of length L with periodic boundary condition. Each species is characterized by a colour q which can be either a, b or c. Particles of the same colour experience a hard core interaction so that not more than one particle of the same colour can occupy a given site. Therefore every site can be either empty or occupied by up to three particles of different colours. Thus there are eight possible states on every site. Let $\hat{q}_i$ and $\hat{q}_i^+$ denote the Pauli operators which annihilate and create a particle of colour q at the site i. Now consider the Hamiltonian

$$\hat{H} = \sum_{i=1}^{L} [\hat{A}_{i,i+1} + \hat{B}_{i,i+1} + \hat{C}_{i,i+1}] \tag{2}$$

where

$$\hat{A}_{i,i+1} = (\hat{b}_i^+ \hat{b}_{i+1}^+ + \hat{c}_i^+ \hat{c}_{i+1}^+ - 2\hat{a}_i^+ \hat{a}_{i+1}^+)\hat{a}_i \hat{a}_{i+1}$$
$$\hat{B}_{i,i+1} = (\hat{a}_i^+ \hat{a}_{i+1}^+ + \hat{c}_i^+ \hat{c}_{i+1}^+ - 2\hat{b}_i^+ \hat{b}_{i+1}^+)\hat{b}_i \hat{b}_{i+1}$$
$$\hat{C}_{i,i+1} = (\hat{a}_i^+ \hat{a}_{i+1}^+ + \hat{b}_i^+ \hat{b}_{i+1}^+ - 2\hat{c}_i^+ \hat{c}_{i+1}^+)\hat{c}_i \hat{c}_{i+1}$$

where the operator $\hat{A}_{i,i+1}$ acting on any state results a non-null vector only when $q_i = q_{i+1} = a$ and is given by

$$\hat{A}_{i,i+1}|\ldots, a, a, \ldots\rangle = |\ldots, b, b, \ldots\rangle + |\ldots, c, c, \ldots\rangle - 2|\ldots, a, a, \ldots\rangle. \tag{3}$$

The operators $\hat{B}_{i,i+1}$ and $\hat{C}_{i,i+1}$ have similar actions. The operator

$$\hat{n}_i = \hat{a}_i^+ \hat{a}_i + \hat{b}_i^+ \hat{b}_i + \hat{c}_i^+ \hat{c}_i \tag{4}$$

counts the number of particles at site i. Clearly

$$[\hat{n}_i, \hat{H}] = 0 \tag{5}$$

and the number of particles is conserved at each site. If we restrict ourselves to the subspace of the full Hilbert space in which there is only one particle at every site, i.e. $\hat{n}_i = 1$ for all i, corresponding to every configuration $|q_1, q_2, \ldots, q_L\rangle$ in this subspace there is a unique configuration specified by the same L-string as in the dimer problem. The action of $\hat{H}$ on $|\{q_i\}\rangle$ is the same as that of $\hat{W}$. Thus $\hat{W}$ is represented by the quantum Hamiltonian given by (2) acting on the subspace with $\hat{n}_i = 1$ for all i.

Make the particle–hole transformation ($\hat{q}_i \to \hat{q}_i^+$) on all odd numbered sites for all q. We define the current operator by

$$\hat{J}_{i,i+1} = \hat{a}_i^+ \hat{a}_{i+1} + \hat{b}_i^+ \hat{b}_{i+1} + \hat{c}_i^+ \hat{c}_{i+1}. \tag{6}$$

Then the Hamiltonian up to the addition of a constant can be written as

$$\hat{H} = \sum_i [\hat{J}_{2i,2i+1} \hat{J}_{2i,2i+1}^+ + \hat{J}_{2i,2i-1} \hat{J}_{2i,2i-1}^+] + 3 \sum_{i,\alpha} n_{i,\alpha} n_{i+1,\alpha}. \tag{7}$$

The dimer model now corresponds to $\hat{H}$ operating on a sector where $\hat{n}_i = 1$ for all even numbered sites and $\hat{n}_i = 2$ for all odd numbered sites. The corresponding stochastic process is one in which there is a constant rate of exchange of any two particles at nearest neighbour sites.

3. The sector decomposition of the vector space

As in the original trimer model, this model has an infinite number of independent constants of motion. These are described most simply in terms of the 'irreducible string' (IS) [3]. We consider free boundary conditions for convenience. Each configuration can be represented as a string of L characters, each character being one of a, b and c. From the string corresponding to a given configuration C, delete all adjacent pairs of aa, bb and cc. Each such deletion decreases the length of the string by 2. Repeat this operation on the resulting string until no more deletions are possible. This defines the IS corresponding to C.

It is straightforward to extend the arguments in [3] to our model, and show that the IS is a constant of motion. In addition, any two configurations having the same IS can be reached from each other. Thus the IS can be used to label the different sectors into which the vector space breaks up.

Consider the set of all configurations in which no adjacent pair of sites are in the same state. These configurations will not evolve under the given dynamics and are said to be fully jammed. For these configurations the length l of the irreducible string equals L. The total number of such configurations is easily seen to be equal to $3 \times 2^{L-1}$. Now consider sectors which are not totally jammed ($l < L$). The number of sectors labelled by irreducible strings of length l will be the number of distinct irreducible strings of length l which is $3 \times 2^{l-1}$. The total number of sectors can easily be seen to be $2^{L+1} - 1$.

Now we proceed to calculate the sizes of these sectors, i.e. the number of distinct configurations which belong to a particular sector. The size of a fully jammed sector is clearly 1. This is because a totally jammed configuration cannot evolve and hence is the only member of that sector. Let $D(IS, L)$ be the size of the sector labelled by irreducible string IS on a lattice of length L. We define the generating function

$$G(IS, z) = \sum_{L=l}^{\infty} D(IS, L) z^L \tag{8}$$

where the summation over L extends from l, the length of IS to infinity. Of special interest is the sector corresponding to the IS of length zero. This we will call the null sector and denote its irreducible string by ϕ. To compute $G(\phi, z)$, we introduce a formal series which is a sum of all strings of arbitrary length, which are made up of three letters a, b and c, and reducible to ϕ:

$$\tilde{G}(\phi) = \phi + aa + bb + cc + aaaa + bbbb + cccc + abba + \cdots. \tag{9}$$

A string S is said to be *decomposable*, if the irreducible string $IS(S) = \phi$ and it can be written as $S_1 \cdot S_2$ such that $IS(S_1) = IS(S_2) = \phi$. If $\tilde{G}_I(\phi)$ denote the sum of all indecomposable strings which are reducible to ϕ then $\tilde{G}(\phi)$ satisfies the equation

$$\tilde{G}(\phi) = \frac{\phi}{1 - \tilde{G}_I(\phi)}. \tag{10}$$

Further if $\tilde{G}_I^q$ denotes the the sum of all indecomposable strings which are reducible to ϕ and starting with the letter q then

$$\tilde{G}_I(\phi) = \tilde{G}_I^a + \tilde{G}_I^b + \tilde{G}_I^c \tag{11}$$

$\tilde{G}_I^q$ is given by

$$\tilde{G}_I^q = q \frac{1}{1 - (\tilde{G}_I^a + \tilde{G}_I^b + \tilde{G}_I^c - \tilde{G}_I^q)} q. \tag{12}$$

To get the generating function we replace all occurrences of a, b, c by z in equation (12). Then $\tilde{G}_l^a$, $\tilde{G}_l^b$ and $\tilde{G}_l^c$ all reduce to the same power series in z, call it $g(z)$, given by

$$g(z) = \frac{z^2}{1-2g(z)}. \tag{13}$$

This equation determines $g(z)$ as an explicit function of z given by

$$g(z) = (1-\sqrt{1-8z^2})/4. \tag{14}$$

Using equations (10) and (11), we get

$$G(\phi, z) = \frac{1}{1-3g(z)}. \tag{15}$$

The growth of $D(\phi, L)$ for large L is determined by the singularities of $g(z)$ nearest to the origin. This happens at $z_c = \pm 1/\sqrt{8}$, and since g has a square root singularity near z_c, it is easily verified that

$$D(\phi, L) \sim 8^{L/2} L^{-3/2} \tag{16}$$

for large L.

One can easily generalize this procedure to find the size of the sector when the irreducible string is of finite length l. Let $IS = \alpha_1\alpha_2 \ldots, \alpha_l$, where $\alpha_i = a$, b or c. Then $\tilde{G}(IS)$ (defined similarly as $\tilde{G}(\phi)$) is given by

$$\tilde{G}(IS) = \frac{1}{1-\tilde{H}(\alpha_1)}\alpha_1 \frac{1}{1-\tilde{H}(\alpha_2)}\alpha_2 \ldots \frac{1}{1-\tilde{H}(\alpha_l)}\alpha_l \tilde{G}(\phi) \tag{17}$$

where $\tilde{H}(\alpha) = \tilde{G}_l^a + \tilde{G}_l^b + \tilde{G}_l^c - \tilde{G}_l^\alpha$. Replacing a, b, c by z as before we get

$$G(IS, z) = \left[\frac{z}{1-2g(z)}\right]^l \frac{1}{1-3g(z)} \tag{18}$$

where $g(z)$ is given by equation (14) and l is the length of the irreducible string IS. Then the size of the sector characterized by the irreducible string IS is the coefficient of z^L in (18), and hence

$$D(IS, L) = \frac{1}{2\pi i}\oint_c dz \frac{G(IS, z)}{z^{(L+1)}} \tag{19}$$

where integration is over a small circle encircling the origin. From this it is straightforward to find the asymptotic behaviour of $D(IS, L)$ by doing the integral using the saddle-point approximation. Let $\lambda = \lim_{L\to\infty} l/L$. Then

$$D(IS, L) \sim [k(\lambda)]^L \tag{20}$$

where

$$k(\lambda) = (1-\lambda)^{-(1-\lambda)/2}(1+\lambda)^{-(1+\lambda)/2}2^{(3-\lambda)/2}. \tag{21}$$

Thus, in general the size of a sector increases exponentially with L and the growth constant depends on the density of the irreducible string. From this one can recover the results for the null and totally jammed sectors by noting that $k(0) = \sqrt{8}$ and $k(1) = 1$.

4. Additional symmetries

In the previous section we discussed the decomposition of the 3^L-dimensional configuration space of the model into $2^{L+1} - 1$ mutually disjoined sectors. This sector decomposition described in terms of the irreducible string IS is maximal in the sense that it is possible to go from any configuration to any other configuration in the same sector. However, this only helps us to block diagonalize $\hat{W}$. In order to do better, we have to find additional operators that commute with $\hat{W}$. These operators are not completely diagonal in the configuration basis. We now describe how to construct such operators using the additional symmetries of our model. These symmetries are not present in the original k-mer model. They are also not present in our model if all the transition rates are not equal. Using these symmetries we are able to reduce the task of diagonalizing the stochastic matrix within a sector, by further block diagonalizing it.

The deposition–evaporation model can be recast in the form of a generalized interface growth model, similar to the KPZ model, where the variable at each site is a 2×2 matrix instead of a scalar†. This is seen as follows. We consider three non-commuting matrices $A(a)$, $A(b)$ and $A(c)$ such that $A^2(a) = A^2(b) = A^2(c) = I_D$, the identity matrix. A simple choice which satisfies these conditions are the unimodular matrices

$$A(a) = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix} \qquad A(b) = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \qquad A(c) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (22)$$

Alternately, we could choose them to be $SU(2)$ unitary matrices. For a configuration $|C\rangle = |q_1, q_2, \ldots, q_L\rangle$, we associate a 2×2 matrix $I_i(C)$ to every site i given by

$$I_i(C) = A(q_i) \ldots A(q_2)A(q_1). \quad (23)$$

It is easy to see that the stochastic evolution of the dimer model can be cast in terms of matrices I's by the following local evolution rule: if $I_{i-1} = I_{i+1}$, then I_i is reset randomly to any one of the three values $A(q)I_{i-1}$ at a constant rate say 1. The conservation law of the IS in this representation becomes the simple statement that I_L is unchanged.

A configuration of dimers is then equivalently characterized by specifying the matrix variables I_i at each site i. This matrix formulation clearly brings out the existence of a discrete group of infinite order in the model. If we work with free boundary conditions the set of allowed values of the of the matrices I's can be put in one-to-one correspondence with the sites of a three-coordinated Bethe lattice. Identify one site of the Bethe lattice as the origin and associate the identity matrix to this site. We colour the bonds of this Bethe lattice by three colours a, b, c such that any three bonds meeting at a site are all of different colours. The matrix I_α corresponding to a site α of the Bethe lattice is given by the ordered product of $A(q)$s ($q = a, b, c$) along the unique path from the origin to α. A sequence $\{q_i\}$, $i = 1$ to L, specifies a configuration of the DDE model, and also a unique L-step path on the Bethe lattice which starts at the origin. As an example, the path corresponding to the configuration *accacc* is shown in figure 1. Configurations in the null sector correspond to paths that return to the origin. Any pair of adjacent spins that are in the same state in the dimer configuration will corresponds to an immediate retraversal of a step on the Bethe lattice. The stochastic evolution in the DDE model leads to a stochastic evolution of this L-step polymer chain on the Bethe lattice. An elementary movement of this polymer chain dynamics is illustrated in figure 1, where the transition from the configuration *acccacc* to *abbacc* is shown.

† Matrix generalizations of the KPZ model, different from this, have recently been discussed in [12, 13].

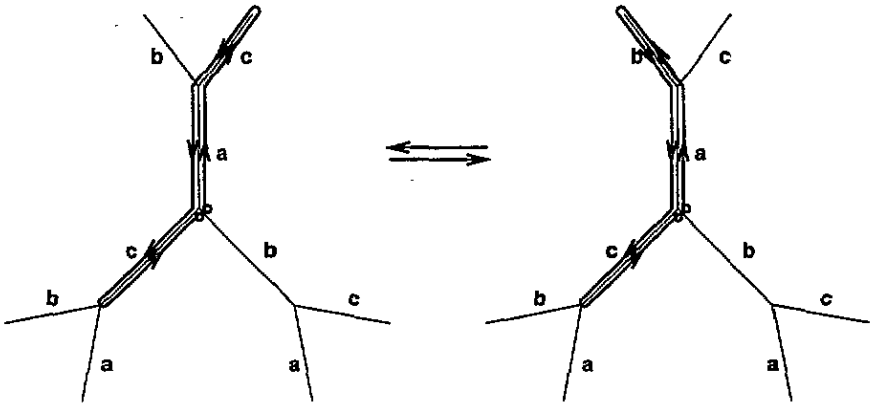

Figure 1. Dimer deposition–evaporation dynamics as a dynamics of polymer chain on the Bethe lattice. The transition between the states $|accacc\rangle$ and $|abbacc\rangle$ is shown.

The operation of interchange of one branch of the Bethe lattice with another is a symmetry of $\hat{W}$ since this only corresponds to a recolouring of the bonds. This is illustrated in figure 2, where we have considered only one branch starting from the origin. At every site α on the Bethe lattice define an operator $\hat{P}_\alpha$ which interchanges the two subtrees starting from that site and going away from the origin. $\hat{P}_\alpha$ changes a configuration $C \equiv \{I_i\}$ to $C' \equiv \{I_i'\}$ where $I_i' \to f(I_i)$, where f is a matrix valued function of its argument.

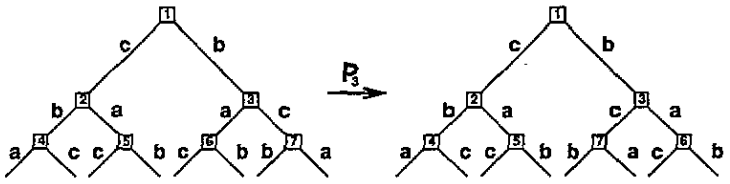

Figure 2. The operation of interchange of two branches that start from the site 3. This is equivalent to recolouring of the bonds, and is a symmetry of $\hat{W}$.

Clearly $[\hat{W}, \hat{P}_\alpha] = 0$. We say that site β is a descendent of site α if the unique path connecting β to the origin goes through α. From the figure it can be seen that $[\hat{P}_\alpha, \hat{P}_\beta] = 0$ if neither α nor β is a descendent of the other. If β is a descendent of α then $\hat{P}_\alpha \hat{P}_\beta = \hat{P}_\gamma \hat{P}_\alpha$, where $\hat{P}_\gamma$ is the operator at site γ which is obtained from site β by the action of $\hat{P}_\alpha$. In figure 2 for example, $[\hat{P}_2, \hat{P}_3] = 0$. But $[\hat{P}_1, \hat{P}_2] \neq 0$ since 2 is a descendent of 1. Instead they satisfy the relation $\hat{P}_1 \hat{P}_2 = \hat{P}_3 \hat{P}_1$. This symmetry comes from the geometrical symmetry of the underlying Bethe lattice. This in turn is related to the internal colour symmetry in the model, and we will call it recolouring symmetry C.

In addition to these one has a permutation symmetry of order 3 (S_3) about the origin which corresponds to interchanging the three colour labels globally. The set of operators $\{\hat{P}_\alpha\}$ together with the operators corresponding to S_3 will constitute an infinite set of conserved quantities, which are, however, not all commuting with each other.

One interesting consequence of the recolouring symmetry is that the spectrum of $\hat{W}$ in a sector depends only on the length l of the irreducible string and not on its details. This can be seen as follows. For a general sector, the endpoints of all the paths on the Bethe lattice will be fixed. The shortest path connecting these two points corresponds to the irreducible

string. All $\hat{P}_\alpha$s such that site α does not belong to this path leave the irreducible string and hence the sector it labels invariant. On the other hand, one can go from one irreducible string to any another one having the same length by a series of branch flip operations using the $\hat{P}_\alpha$ operators which are on the shortest path. Since each of these operations are symmetry operations of $\hat{W}$, i.e. $[\hat{P}_\alpha, \hat{W}] = 0$ for all α, the spectrum will be identical for all these sectors. This property is not present in the original k-mer model.

We can make use of these symmetries for further block diagonalization of $\hat{W}$ with in a sector. This is achieved by breaking the state space spanned by all the configurations in a particular sector into subspaces. We illustrate this with a few examples.

For convenience we will consider the case of null sector. First consider a lattice of length 2. There are only three configurations in this sector which are $|aa\rangle$, $|bb\rangle$ and $|cc\rangle$. In this case we can get all the eigenstates of the Hamiltonian using S_3. Consider the states

$$\begin{aligned} |\psi_1\rangle &= |aa\rangle + |bb\rangle + |cc\rangle \\ |\psi_2\rangle &= |aa\rangle + \omega|bb\rangle + \omega^2|cc\rangle \\ |\psi_3\rangle &= |aa\rangle + \omega^2|bb\rangle + \omega|cc\rangle \end{aligned} \tag{24}$$

where ω is the cube root of unity. These are the eigenstates of the operator corresponding to cyclic permutation of a, b and c ($\hat{P}_{abc}$) with eigenvalues 1, ω and ω^2 respectively. Since $[\hat{P}_{abc}, \hat{W}] = 0$ these are also the eigenstates of $\hat{W}$. The eigenvalues of $\hat{W}$ corresponding to these eigenvectors are $0, -3, -3$. Thus in this case we were able to diagonalize $\hat{W}$ completely. But this is a rather trivial example.

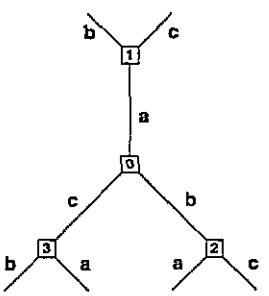

Figure 3. Construction of exact eigenvectors using recolouring symmetry. For $L = 4$, three exact eigenvectors can be constructed by antisymmetrizing about the sites 1, 2 and 3.

Next we consider the case of $L = 4$. There are 15 states in this sector. First we consider the symmetry operations about the sites 1, 2 and 3 as shown in figure 3. Consider the states

$$\begin{aligned} |\psi_1\rangle &= \tfrac{1}{\sqrt{2}}(|abba\rangle - |acca\rangle) \\ |\psi_2\rangle &= \tfrac{1}{\sqrt{2}}(|bccb\rangle - |baab\rangle) \\ |\psi_3\rangle &= \tfrac{1}{\sqrt{2}}(|caac\rangle - |cbbc\rangle). \end{aligned} \tag{25}$$

These are the non-degenerate eigenstates with eigenvalue -1 of $\hat{P}_1$, $\hat{P}_2$ and $\hat{P}_3$ respectively. It can be verified very easily that these are also eigenstates of $\hat{W}$. This gives us a decomposition of the 15-dimensional vector space into subspaces of dimensions 1, 1, 1 and 12. Using the cyclic permutation symmetry at the origin, we can get a further decomposition of the 12-dimensional subspace. For this consider the eigenstates of $\hat{P}_{abc}$

$$|\phi_1\rangle = \tfrac{1}{\sqrt{3}}(|aaaa\rangle + |bbbb\rangle + |cccc\rangle)$$
$$|\phi_2\rangle = \tfrac{1}{\sqrt{3}}(|aabb\rangle + |bbcc\rangle + |ccaa\rangle)$$

$$|\phi_3\rangle = \tfrac{1}{\sqrt{3}}(|bbaa\rangle + |ccbb\rangle + |aacc\rangle)$$
$$|\phi_4\rangle = \tfrac{1}{\sqrt{6}}(|abba\rangle + |acca\rangle + |bccb\rangle + |baab\rangle + |caac\rangle + |cbbc\rangle)$$

$$|\chi_1\rangle = \tfrac{1}{\sqrt{3}}(|aaaa\rangle + \omega|bbbb\rangle + \omega^2|cccc\rangle)$$
$$|\chi_2\rangle = \tfrac{1}{\sqrt{3}}(|aabb\rangle + \omega|bbcc\rangle + \omega^2|ccaa\rangle)$$
$$|\chi_3\rangle = \tfrac{1}{\sqrt{3}}(|bbaa\rangle + \omega|ccbb\rangle + \omega^2|aacc\rangle)$$
$$|\chi_4\rangle = \tfrac{1}{\sqrt{6}}((|abba\rangle + |acca\rangle) + \omega(|bccb\rangle + |baab\rangle) + \omega^2(|caac\rangle + |cbbc\rangle))$$

$$|\xi_1\rangle = \tfrac{1}{\sqrt{3}}(|aaaa\rangle + \omega^2|bbbb\rangle + \omega|cccc\rangle)$$
$$|\xi_2\rangle = \tfrac{1}{\sqrt{3}}(|aabb\rangle + \omega^2|bbcc\rangle + \omega|ccaa\rangle)$$
$$|\xi_3\rangle = \tfrac{1}{\sqrt{3}}(|bbaa\rangle + \omega^2|ccbb\rangle + \omega|aacc\rangle)$$
$$|\xi_4\rangle = \tfrac{1}{\sqrt{6}}((|abba\rangle + |acca\rangle) + \omega^2(|bccb\rangle + |baab\rangle) + \omega(|caac\rangle + |cbbc\rangle)). \tag{26}$$

Since $\hat{P}_{abc}$ commutes with $\hat{W}$, $\{|\phi_i\rangle\}$, $\{|\chi_i\rangle\}$ and $\{|\xi_i\rangle\}$ form three invariant subspaces under the action of $\hat{W}$. Thus we have got the following decomposition of the 15-dimensional vector space spanned by all the configurations in the null sector in to subspaces of dimensions

$$1 + 1 + 1 + 4 + 4 + 4 = 15. \tag{27}$$

We have carried out the same procedure for the case of $L = 6$. In this case the size of the null sector is 87. Using the $\hat{P}_\alpha$ operators, this can be broken into 18 subspaces out of which 12 are of size 1, 3 are of size 7 and another 3 are of size 18.

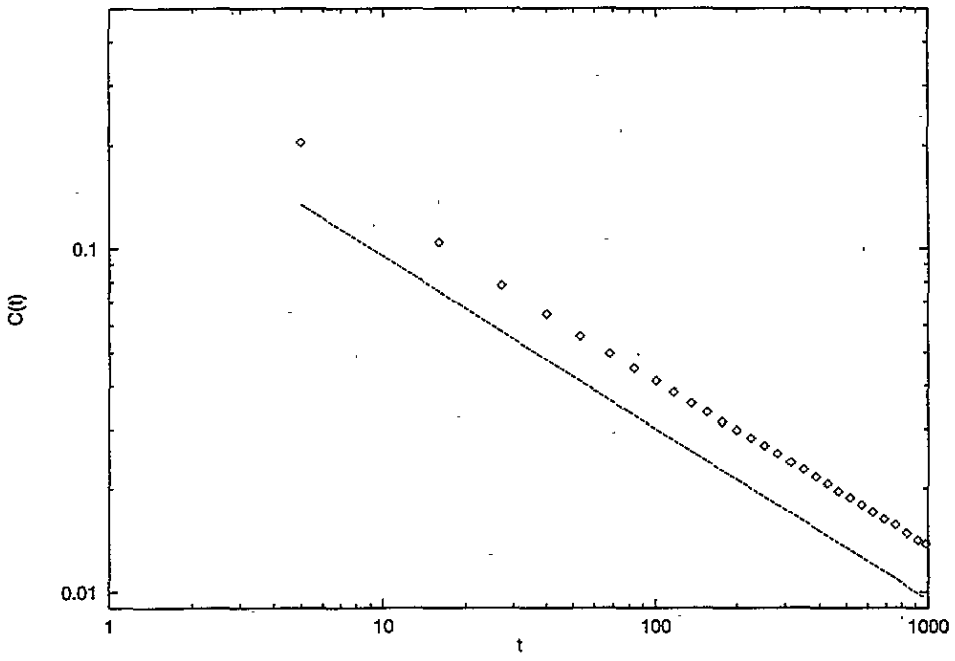

Figure 4. The autocorrelation function $C^A_{aa}(t)$ for the sector $abab\ldots$ (◇). The broken curve is a plot of $t^{-1/2}$.

form of the decomposition of the vector space for the case $L = 6$ is

$$87 = 12 \times 1 + 3 \times 3 + 3 \times 4 + 3 \times 6 + 3 \times 12 \tag{28}$$

where $m \times n$ means m subspaces of size n.

In principle this procedure can be carried out for a lattice of any length. For a linear chain of length L, sites up to distance $L/2$ from the origin on the Bethe lattice are reachable. The number of sites that are one step away from the boundary and inside will be $3 \times 2^{(L-4)/2}$ ($L \geqslant 4$). As we have seen for $L = 4$, the antisymmetric eigenstates of the $\hat{P}$ operators on each of these sites will be an eigenstate of $\hat{W}$. Thus one can easily get a large number of eigenstates in this model. By constructing the antisymmetric eigenstates of the $\hat{P}$ operators on sites which are interior to this, one can get invariant subspaces as we have seen in the case of $L = 6$ (the three sectors of size 7). So far we have not been able to get a general result for the total number of subspaces and their sizes within a sector as a result of this symmetry. But one can use this symmetry for the numerical diagonalization of the stochastic matrix for small sizes, where one can restrict oneself to one subspace (say the symmetric subspace where all the eigenvalues of the $\hat{P}$ operators are 1). This we have done to estimate the dynamical exponent in the null sector, where we have seen that these additional symmetries lead to substantial decrease in the size of the matrices to be diagonalized. These results are discussed in section 8.

5. Construction of some exact eigenvectors of the stochastic matrix

In the previous section we have constructed some of the eigenvectors of the stochastic matrix in the null sector using the recolouring symmetry of the dynamics of L-step chain on the Bethe lattice. In this section we show that the same procedure can be used to construct eigenvectors in sectors which are almost totally jammed, i.e. the length of the irreducible string $l = L - 2n$, where n is a small integer.

As we explained in the earlier section, the spectrum of $\hat{W}$ is the same for all the sectors whose irreducible strings are of same length. So we need to consider only one representative sector which we choose to be the one in which the irreducible string is *abab*.... Consider the case when $l = L - 2$. We use periodic boundary condition for convenience. There are two types of configurations in this sector. In the first type there will be three adjacent sites, say $i, i+1$ and $i+2$ which are of the same colour (either *aaa* or *bbb* e.g. |...*abbbab*...⟩). Let these states be denoted by $|\phi(i)\rangle$. In the second type there will be two adjacent sites, say $i+1$ and $i+2$ which are having colour c. (e.g. |...*abccab*...⟩.) Let this type of states be denoted by $|\chi(i)\rangle$. In both these types site $i+1$ and site $i+2$ have the same colour and thus constitute a reducible part. It can be very easily seen that under the deposition–evaporation dynamics, the position of the reducible part makes a nearest-neighbour random walk. Hence the dynamics in this sector is equivalent to a random walk problem with two states per site ($|\phi(i)\rangle$ and $|\chi(i)\rangle$). The eigenfunctions $|\psi_k\rangle$ and eigenvalues λ_k of the stochastic matrix can be found by solving this random walk problem.

Let $\phi_k(i)$ and $\chi_k(i)$ be defined by $\phi_k(i) = \langle\phi(i)|\psi_k\rangle$ and $\chi_k(i) = \langle\chi(i)|\psi_k\rangle$. They satisfy the following eigenvalue equations

$$\begin{aligned}\lambda_k\phi_k(i) &= \phi_k(i-1) + \phi_k(i+1) + \chi_k(i-1) + \chi_k(i) - 4\phi_k(i)\\ \lambda_k\chi_k(i) &= \phi_k(i) + \phi_k(i+1) - 2\chi_k(i).\end{aligned} \tag{29}$$

The solution for these eigenvalue equations are given by

$$\lambda_k = (3 - \cos(k)) \pm \sqrt{3 + \cos^2(k)} \tag{30}$$

and

$$\phi_k(n) = e^{ikn}$$
$$\chi_k(n) = \frac{1}{\lambda_k + 2}(e^{ikn}(1 + e^{ik})) \tag{31}$$

where $k = 2\pi m/L$, $m = 0, 1, \ldots, L-1$.

In the case of free boundary conditions some of the localized eigenvectors can be constructed directly with out solving this random walk problem. Consider the situation where the reducible part is near one of the boundaries, i.e. the states $|ab\ldots abaa\rangle$, $|ab\ldots abbb\rangle$ and $|ab\ldots abcc\rangle$. It is easy to see that the state obtained by antisymmetrizing, $|\psi\rangle = |ab\ldots abaa\rangle - |ab\ldots abcc\rangle$, is an eigenstate with eigenvalue -1. One more eigenstate can be constructed by antisymmetrizing about the other end.

Now consider the case of $l = L - 4$. In this case the reducible part is of length 4. A typical configuration in this sector will be $|\ldots ab\alpha\alpha ab\ldots ab\beta\beta ab\ldots\rangle$. The reducible parts $\alpha\alpha$ and $\beta\beta$ perform random walk under the dynamics. When these random walkers come closer to each other they can form a state like $|\ldots ab\alpha\alpha\alpha\alpha ab\ldots\rangle$ or an intermediate state like $|\ldots abc\beta\beta cab\ldots\rangle$. There are two such intermediate states which corresponds to the two values $\beta = a, b$. The two random walkers can go into such intermediate states and remain there for a while by making a transition between them. This can be considered as a short-range attractive interaction between the two walkers. So the dynamics in this sector is equivalent to a problem of two random walkers on a line having a short range attractive interaction.

We can use the same procedure of antisymmetrization to get some more of the eigenvectors. One type of eigenvector is obtained by antisymmetrizing about both ends simultaneously:

$$|\psi_1\rangle = [|bbab\ldots abaa\rangle - |bbab\ldots abcc\rangle] - [|ccab\ldots abaa\rangle - |ccab\ldots abcc\rangle]. \tag{32}$$

The second type of eigenvector is obtained by antisymmetrizing two intermediate states $|\ldots abcaacab\ldots\rangle$ and $|\ldots abcbbcab\ldots\rangle$:

$$|\psi_2\rangle = |\ldots abcaacab\ldots\rangle - |\ldots abcbbcab\ldots\rangle. \tag{33}$$

Since there are L positions on the string, where $abc\beta\beta cab$ can occur, for the case of periodic boundary conditions, there are L such eigenvectors.

This can easily be generalized for the case where there are $2n$ random walkers ($2n < L$). A typical intermediate state formed by pair wise union of random walkers will have n number of substrings of the form $\ldots abc\alpha\alpha cab\ldots$. The eigenvector is constructed by antisymmetrization of all these. The corresponding eigenvalue will be $-3n$. Since these n substrings can be arranged in roughly ${}^{L}C_n$ ways, we can get so many eigenvectors. As an example, in the case of $n = 2$ one such eigenvector is given by

$$|\psi\rangle = |\ldots abcaacab\ldots abcaacab\ldots\rangle - |\ldots abcbbcab\ldots abcaacab\ldots\rangle$$
$$+|\ldots abcbbcab\ldots abcbbcab\ldots\rangle - |\ldots abcaacab\ldots abcbbcab\ldots\rangle. \tag{34}$$

6. Autocorrelation functions

For the trimer model, the existence of the infinite number of conservation laws leads to non-exponential relaxation in equilibrium [4]. It is found that the density–density autocorrelation function decay as a power law for large times and the exponent is different in different sectors. In most sectors the decay is $t^{-1/4}$ but is of the form $t^{-1/2}$ in most of the sectors with periodic irreducible strings. In some special sectors with periodic irreducible string,

stretched exponential behaviour ($\exp(-\sqrt{t})$) is found. In the null sector the autocorrelation function is found to decay as $t^{-3/2z}$, where $z \approx 2.5$. This diversity of relaxational behaviour has been explained in terms of the hard core random walkers with conserved spins (HCRWCS) model [4, 6]. It has been argued on qualitative grounds that these two models are in the same universality class. However, a strict proof of this proposition is not yet available.

An important assumption of the HCRWCS model is that the spin carried by each walker is conserved, but does not affect the diffusive motion of the walkers in any way. This is not strictly true in the trimer model. However, for our model, this property can be exactly established as a consequence of the recolouring symmetry. Thus the argument which predicts the different behaviour of spin–spin autocorrelation function in different sectors is much cleaner for this model than for the trimer model studied earlier. We proceed to give the arguments which are an adaptation of the arguments for the trimer deposition evaporation model [4, 6], in some detail. The time-dependent spin–spin autocorrelation function in the steady state is defined by

$$C^i_{\alpha\beta}(t) = \text{Prob}(q_i(t) = \beta, q_i(0) = \alpha) - \text{Prob}(q_i(t) = \beta)\text{Prob}(q_i(0) = \alpha) \tag{35}$$

where Prob(x) denote the probability of event x and $q_i(t)$ is the state of the spin at the ith site at time t. Each of α and β is either a, b or c. Since α and β each takes three values, one can define nine autocorrelation functions in this model. Among these, by time reversal symmetry $C^i_{\alpha,\beta}(t) = C^i_{\beta,\alpha}(t)$ and also $\sum_\beta C^i_{\alpha,\beta}(t) = 0$. Thus there are only three independent correlation functions.

As in the trimer model these autocorrelation functions show sector-dependent decay [4]. Let us consider the following five different representative sectors characterized by the irreducible strings: (i) $ababab\ldots$, (ii) $acbcacbc\ldots$, (iii) $abcabc\ldots$, (iv) random string, and (v) ϕ (null sector), where $\ldots$ denotes repetition. The length of the irreducible string in the first four cases is taken to be $L/2$.

We will first present a qualitative theory for the decay of autocorrelation functions and then compare its predictions with our Monte Carlo data. The behaviour of the autocorrelation function can be understood in terms of the random walk of the characters which constitute the irreducible string. Consider a sector labelled by an irreducible string $\alpha_1, \alpha_2 \ldots, \alpha_l$, with the length of the irreducible string l a finite fraction of the total length L. Let $x_1, x_2 \ldots, x_l$ be the positions of the characters in a given configuration C which do not get deleted under the deletion algorithm of section 3. Then $q_{x_i}(t) = \alpha_i$ ($i = 1$ to l) at all times. We can think of $x_1, x_2 \ldots, x_l$ as the positions of l interacting random walkers with the nth random walker from the left carrying a colour $\alpha_n = q_{x_n}$ with it. The positions of random walkers, $\{x_i\}$ will change in time as C changes. The walkers move either to the left or right but always remain on the same sublattice. Further, they do not cross each other.

We have argued in section 4 that the spectrum of the stochastic matrix in this model is completely independent of the detail of the irreducible string string sequence $\{\alpha_i\}$ and only depend upon the length of the sequence. However, the autocorrelation involves a weighted sum of correlations of $\{\alpha_i\}$, which gives rise to different relaxational behaviour in different sectors.

The important contribution to the autocorrelation function comes from times when the site i is occupied by a character from the irreducible string. We shall assume without proof that the contribution coming from times when the site i is occupied by a reducible character is qualitatively similar.

Since the hard core random walkers will remain on the same sublattice they were initially, we can perform a sublattice average of $C^i_{\alpha,\beta}$. Let Γ denotes the sublattice which is either A (odd sites) or B (even sites). In the HCRWCS approximation we can write the

expression for the autocorrelation function as

$$C^{\Gamma}_{\alpha\beta}(t) \cong \sum_{k=-\infty}^{+\infty} P(k,t) M^{\Gamma}(k) \tag{36}$$

where $P(k,t)$ gives the joint probability of finding two particles at a given site, one at time 0 and the other at time t such that the difference between their labels is $2k$. $M^{\Gamma}(k)$ is a measure of the correlation of the colours in the irreducible string, and is given by

$$M^{\Gamma}(k) = \sum_{m} \delta(q_{x_m}, \alpha)\delta(q_{x_{m+2k}}, \beta) \tag{37}$$

where the summation is over all the labels of the particles which belong to the sublattice Γ. The probability $P(k,t)$ can be obtained using the results of random walks of hard core particles [7–9].

Note that $P(k,t)$ depends only on the total length of the irreducible string and not on its details. While $M^{\Gamma}(k)$ depends only on the irreducible string and is independent of time. This separation of the sector dependence and time dependence is the crucial feature which allows calculation of $C^{\Gamma}_{\alpha,\beta}$ in different cases. *This separation is exact due to the recolouring symmetry in our model, but only approximate in the original trimer model.*

Equation (36) can be used to find the behaviour of autocorrelation function. For example consider the sector labelled by the irreducible string $abab\ldots$. On the A sublattice $q_{x_m} = a$ for all m and hence

$$C^{A}_{aa}(t) = \sum_{k=-\infty}^{+\infty} P(k,t). \tag{38}$$

This is the density–density correlation function of hard core particles which decays as $t^{-1/2}$ for large t [7]. Our simulation gives the same power law decay which is shown in figure 4. From the symmetry between b and c on this sublattice and the constraint $\sum_{\beta} C^{\Gamma}_{\alpha,\beta}(t) = 0$, it can be easily seen that $C^{A}_{\alpha,\beta}(t) \sim t^{-1/2}$ for all values of α and β. The behaviour of the correlation functions on both the sublattices is the same which follows from the symmetry of the irreducible string.

In the case of the sector labelled by the irreducible string $acbc\ldots$, $M^{A}(k)$ is of the form $c_1 + c_2(-1)^k$, where c_1 and c_2 are constants independent of k. Thus $C^{A}_{aa}(t)$ has two parts. The first part decays like $t^{-1/2}$ as explained earlier. The second part is the Fourier transform of $P(k,t)$ which goes as $\exp(-\sqrt{t})$ for large t. The stretched exponential decay dominates the short term behaviour but asymptotically the behaviour will cross over to $t^{-1/2}$. By symmetry, $C^{A}_{bb}(t)$ should have the same decay. On the B sublattice q_{x_m} is c for all m, so the decay will be purely diffusive ($t^{-1/2}$) as explained earlier. These behaviours can be seen in figure 5.

For the sector $abcabc\ldots$, the decay of $C^{A}_{aa}(t)$ is shown in figure 6. The leading behaviour of the correlation function can be shown to be stretched exponential decay by a similar argument as in the previous case. By symmetry of the irreducible string the behaviour should be the same for all the correlation functions and on both the sublattices. This has been confirmed in our simulation.

When the irreducible string is random, $M^{\Gamma}(k)$ is significant only for small values of k. For small k, $P(k,t) \sim t^{-1/4}$ for large t [4]. Therefore the autocorrelation has a $t^{-1/4}$ decay which can be seen in figure 7.

Figure 7 also shows the decay of $C^{A}_{aa}(t)$ in the null sector. It is a power law decay with the value of the exponent ≈ 0.59. The same power law decay is found for other values of α and β and on both sublattices, which follows from symmetry. Barma *et al* have obtained the

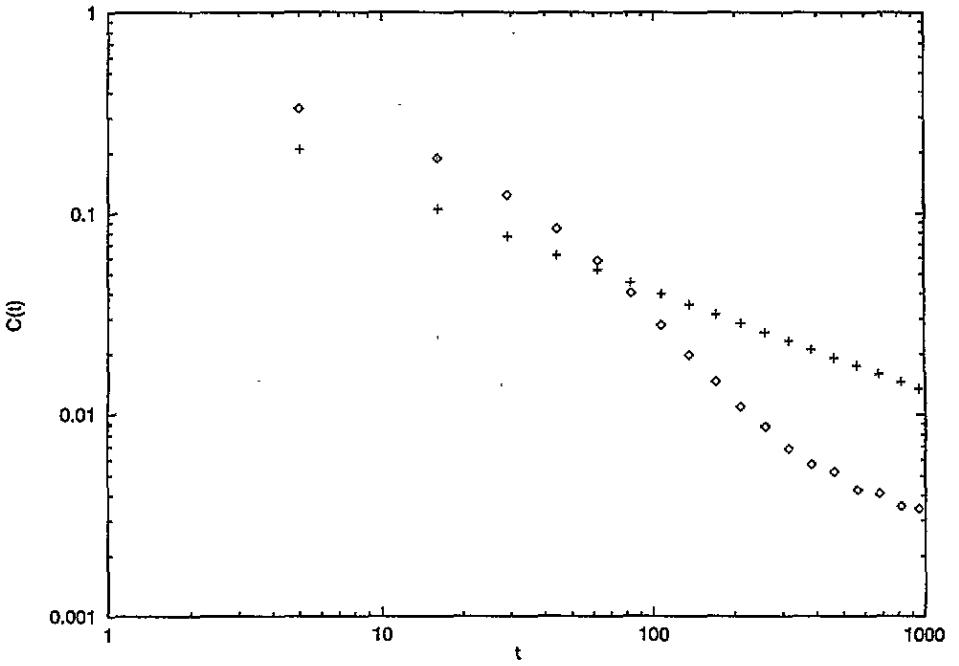

Figure 5. The autocorrelation functions $C^A_{aa}(t)$ ($\diamond$) and $C^B_{bb}(t)$ (+) for the sector *acbc* .... For small t, $C^A_{aa}(t)$ has a stretched exponential decay of the form $\exp(-t^{-1/2})$ but at later times it crosses over to pure diffusive decay. On the other hand $C^B_{aa}(t)$ has pure diffusive decay at all times.

same estimate for this exponent for the trimer model in the null sector [4]. This is evidence that the dynamics in these two models for the case of the initial condition in which all sites are in the same state belongs to the same universality class. We can understand the behaviour of the correlation functions in sectors where the irreducible string has a finite density using the HCRWCS model, this approach is no longer useful when studying the null sector. An analytical understanding of the temporal decay of autocorrelation functions in this sector is still lacking.

7. Spatial correlation function in the steady state

Let $C_{\alpha,\beta}(r)$ be the probability that, in the steady state, spins at two sites which are separated by a distance r are α and β. We calculate this equal time spatial correlation function for the null sector. Let us assume periodic boundary conditions. Let S be the string corresponding to a given configuration on this lattice with sites i and $i+r$ occupied by α and β. Let S_1 be the substring of length $r-1$ formed by spins between α and β and S_2 be the substring of length $L-r+1$ formed by the remaining spins. The endpoints of the substring S_2 are α and β. Let $IS(S_1)$ and $IS(S_2)$ be the irreducible strings corresponding to S_1 and S_2. Since $IS(S) = \phi$, $IS(S_2)$ will be $IS(S_1)$ reflected about one end. We may write $IS(S_2) \equiv IS(S_1)^{-1}$.

$$C_{\alpha,\beta}(r) = \frac{\sum_{IS} D(IS, r-1) D_{\alpha,\beta}(IS^{-1}, L-r+1)}{D(\phi, L)} \qquad (39)$$

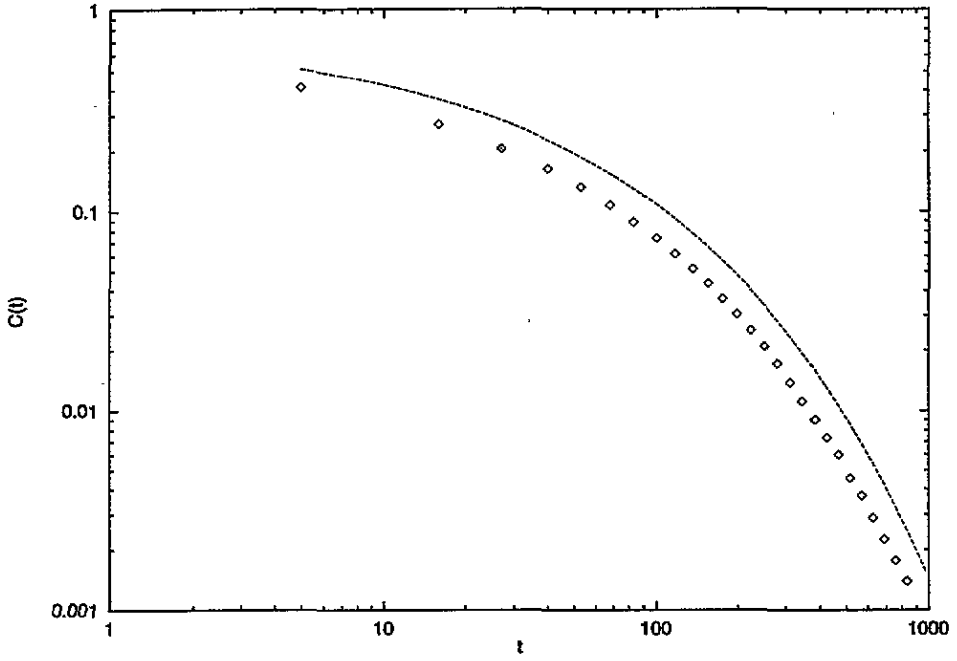

Figure 6. The autocorrelation function $C^{A}_{aa}(t)$ for the sector *abcabc*.... The decay is stretched exponential of the form $\exp(-t^{-1/2})$ which shown by the broken curve.

where the summation is over all the distinct irreducible strings of S_1. $D(IS, r-1)$ is the size of the sector labelled by IS on a lattice of length $r-1$ with free boundary conditions and $D_{\alpha,\beta}(IS^{-1}, L-r+1)$ is the size of the sector labelled by IS^{-1} on a lattice of length $L-r+1$ with boundary sites fixed at α and β.

Though we can calculate exact expressions for each of the above quantities, we derive the asymptotic behaviour by taking $L \to \infty, r << L$. The length of the irreducible string $l \leqslant r$, therefore in this limit one can make the approximation $D_{\alpha,\beta}(IS, L-r+1) \approx D_{\alpha,\beta}(\phi, L-r-l+1)$. In the case of free boundary condition the size of a sector depends only on the length of the irreducible string (21), (22) and we may replace the sum over distinct irreducible strings by a sum over the lengths of the irreducible strings as follows:

$$C_{\alpha,\beta}(r) = \frac{\sum_l N(l) D(l; r-1) D_{\alpha,\beta}(\phi, L-r-l+1)}{D(\phi, L)} \tag{40}$$

where $N(l)$ is the number of distinct irreducible strings of length l, which is given by $3 \times 2^{l-1}$. For large L, $D_{\alpha,\beta}(\phi, L-r-l+1) \sim (2\sqrt{2})^{L-r-l+1}/(L-r-l+1)^{3/2}$. Hence in the summation over l only terms corresponding to small l will contribute. We can find the asymptotic r dependence by approximating the sum by only the $l=0$ term. Then we have

$$C_{\alpha,\beta}(r) \sim D(\phi, r-1) D_{\alpha,\beta}(\phi, L-r+1)/D(\phi, L). \tag{41}$$

For large L this gives

$$C_{\alpha,\beta}(r) \sim \frac{f(\alpha,\beta)}{r^{3/2}} \tag{42}$$

for large r, where $f(\alpha,\beta)$ is a constant independent of r but depends on α and β. From the symmetry between different colours, it follows that $f(\alpha,\beta) = k(1-3\delta_{\alpha,\beta})$.

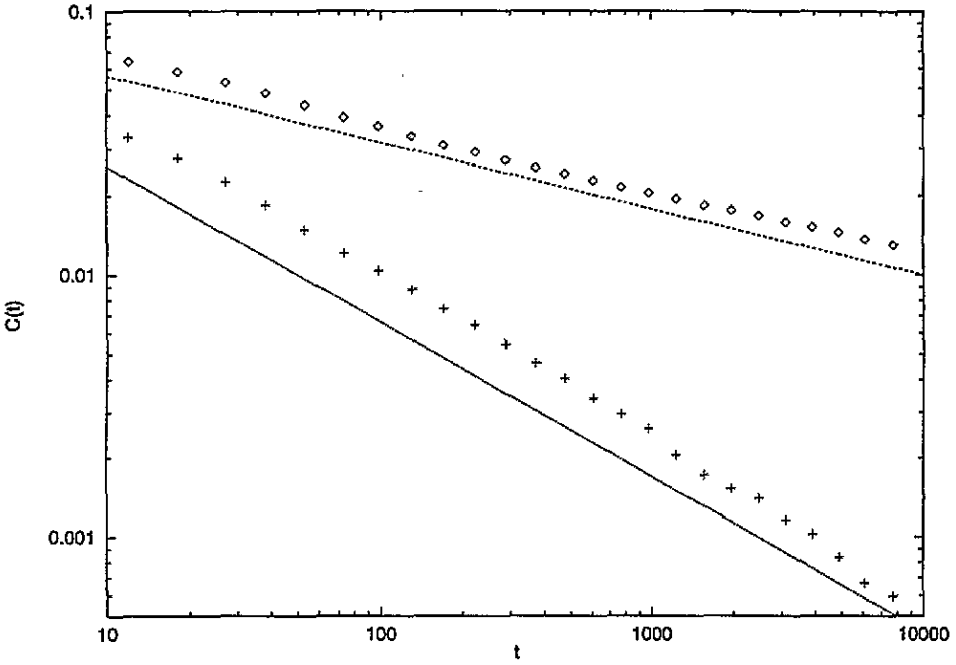

Figure 7. The decay of $C^A_{aa}(t)$ (◇)in the sector where irreducible string is random, and in the null sector (+). These decays are approximately given by $t^{-1/4}$ (broken curve), and $t^{-0.59}$ (dotted curve).

8. Numerical diagonalization of the stochastic matrix in the null sector

As mentioned in section 6, the decay of autocorrelation function in the null sector suggests that the dynamics in this sector belongs to a new universality class. The description of dynamics in terms of the random walk of the characters of the irreducible string is not possible in this sector. One way of studying this sector is to directly diagonalize the stochastic matrix for small sizes. However, this task is not very easy because the size of the matrix to be diagonalized grows very fast with the size of the lattice ($\sim (2\sqrt{2})^L/L^{3/2}$). One can make use of various symmetries of the model to block diagonalize the matrix first and hence reduce the size of the matrix to be diagonalized. By using translation, reflection and the recolouring symmetry described in section 4 we have been able to diagonalize the stochastic matrix for lattice sizes up to 20, in the fully symmetric subspace. The algorithm used here for diagonalizing the matrix using the various symmetries is similar to the one we have used for studying the trimer model [10]. For details of the algorithm we refer the reader to this reference.

In table 1, we have listed the size of the matrix ($N_{R,T,C}$) obtained after using all the three symmetries and the value of the largest non-zero eigenvalue in the symmetric subspace in the null sector for lattice sizes L from 2 to 20. For comparison we also list the total size (N) of the null sector, and also the reduced matrix size (N_C) if only the recolouring symmetry C is taken into account. We see that the additional recolouring symmetry gives a significant reduction in the size of the matrix to be diagonalized.

The largest eigenvalue of the stochastic matrix is 0. The difference between the largest and the second largest eigenvalues gives the gap in the spectrum of the stochastic matrix.

Table 1.

L	N	N_C	$N_{R,T,C}$	λ_L	z_L
2	3	1	1		
4	15	3	2	−10.000 00	
6	87	12	4	−3.108 548	2.881 701
8	543	55	10	−1.629 235	2.245 691
10	3543	271	26	−0.974 243	2.304 369
12	23 823	1399	93	−0.632 500	2.369 330
14	163 719	7470	338	−0.438 272	2.379 750
16	1143 999	40 931	1474	−0.318 921	2.380 690
18	8099 511	228 918	6801	−0.240 934	2.380 807
20	57 959 535	1301 778	33 746	−0.187 477	2.381 040

We have computed this gap for lattice sizes from 2 to 20. By using the finite size scaling relation $\Delta_L \sim L^{-z}$, the dynamical exponent z_L for a lattice of size L is estimated from the relation

$$z_L = \log\left(\frac{\lambda_L}{\lambda_{L-2}}\right) \Big/ \log\left(\frac{L-2}{L}\right). \tag{43}$$

Estimates of z_L for various values of L are shown in table 1. The convergent of z_L seemed to be quite good, and the estimate of the extrapolated value of z corresponding to $L = \infty$ is 2.39 ± 0.05. The error bar reflects our subjective estimate of the possible systematic errors in the extrapolation. This value is in good agreement with the value of $z = 2.5 \pm 0.15$ for the trimer model in the null sector [10], and is evidence that both these models are in the same universality class.

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